Comparison of Belgian pension funding methods using exchange option prices

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The challenge of pensions

The Belgian pension system has been facing several tensions regarding both its financial and social sustainability.

Among various responses to this challenge, the Belgian public authorities have recently (December 2015) adopted an act concerning the occupational (a.k.a. second pillar) pension schemes.

We focus here on the newly implemented computation method that sponsors (employers) of occupational pension plans with defined contributions must apply.

A minimum guaranteed rate

The new act is designed as a reform of the Belgian Law on Complementary Pension (LCP, April 2003).

The LCP was imposing to sponsors to ensure a minimum financial performance on plan contributions: they were legally responsible for guaranteeing a minimum return of 3.25% on the employer's contributions and of 3.75% on employees' contributions.

Such a rate was consistent with the economical context of the adoption of the LCP, the 10-year yield rate on Belgian governmental bonds being at that time above the 4% level.

However, this guaranteed rate was fixed and proved to be unsustainable, in a context of decreasing market rates of return. Indeed, market yields have heavily decreased since the adoption of the LCP, reaching very low levels in 2015.

New definition of the minimum rate

The reform act therefore transforms the fixed minimum guaranteed rate into a *variable* rate, depending on the market conditions through the OLO rate (yield rate of the Belgian governemental 10-year bonds, *Obligations Linéaires - Lineaire Obligaties*).

It is defined as a capped and floored 24-month moving average:

$$
\begin{aligned}
r_{\text{guaranteed}}(t) &= \begin{cases} 1.75\% & \text{if } r_{\text{market}}(t) \leq 1.75\% \\ r_{\text{market}}(t) & \text{if } 1.75\% < r_{\text{market}}(t) < 3.75\% \\ 3.75\% & \text{if } r_{\text{market}}(t) \geq 3.75\% \end{cases} \\
&= \min(1.75\%; \max(r_{\text{market}}(t); 3.75\%)),
\end{aligned}
$$

where

$$
r_{\text{market}}(t) = 65\% \cdot \frac{1}{24} \sum_{s=0 \text{ months}}^{23 \text{ months}} r_{\text{OLO 10}}(t-s).
$$

Historical evolution of the rates

The horizontal and vertical methods

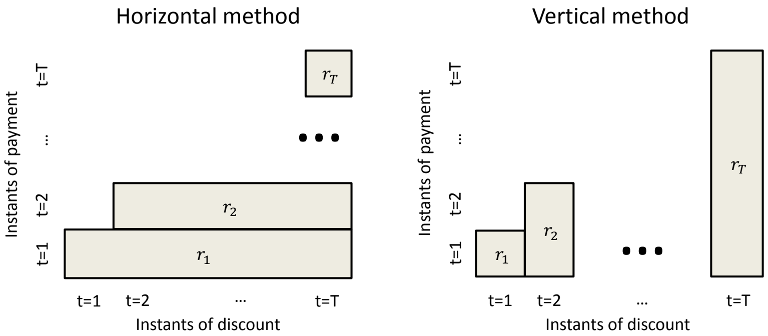
As the new guaranteed rate is variable, one therefore needs to indicate how the guarantee is applied to the precedingly paid contributions.

Two different computation methods are described in the new legal text. The choice between them depends on the funding vehicle of the pension scheme if it already exists at 01/01/2015, and on the choosing of the pension sponsor if the pension scheme is created after 01/01/2015.

The **vertical method** is used when the existing pension scheme is financed through a pension fund (or an "insurance fund") or when the sponsor of the new pension scheme chooses so. In this case, the guaranteed rate of year t is applied to the whole amount of contributions already paid by the affiliated up to year t. The *vertical capital* is then similar to the one produced by a saving account where the contributions are deposited, whose interest rate is the guaranteed rate, susceptible to be different from year to year.

The horizontal and vertical methods

The **horizontal method** is used when the existing pension scheme is funded through a “standard” insurance product (with a fixed guaranteed rate) or when the sponsor of a new pension scheme chooses so. The guaranteed rate of year t is then applied to the new contributions paid in year t, the contributions already paid in years $t-1$ being accounted with the guaranteed rate of year $t-1$, etc. The resulting *horizontal capital* is similar to the one produced by a standard life insurance products.



A very simple example

Let us first take a very simple example: assume that an individual pays two contributions of 1€ in 2016 and 2017, and retires in 2018.

Consider two scenarios for the evolution of the guaranteed interest rate:

	Scenario 1	Scenario 2
2016 guaranteed rate 2017 guaranteed rate	2.5% 3.5%	2.5% 2%
2018 capital (horizontal)	$1.025^2 + 1.035$ $= 2.086$	$1.025^2 + 1.02$ $= 2.071$
2018 capital (vertical)	$1.025 \cdot 1.035 + 1.035$ $= 2.096$	$1.025 \cdot 1.02 + 1.02$ $= 2.066$
Highest capital	vertical	horizontal

Stochastic setting

In order to compare the two methods, we define the following setting.
We assume that the short rate r_t follows a Vasicek dynamics in the risk neutral setting

$$dr_t = k(\theta - r_t)\,dt + \sigma\,dW_t^r.$$

where $k > 0$, $\sigma > 0$ and θ are real constants and W^r is a standard Brownian motion.

In order to obtain closed formula, we make three assumptions:

1. We replace the coupon-bearing reference bond instrument by a standard zero-coupon bond,
2. We define the reference rate R as a 3 years moving average of the market rate (instead of a 24 months moving average),
3. We neglect the cap and floor applied to the market rate.

The guaranteed rate is thus equal to (using $\pi = 65\%$)

$$R_t = \pi\ \frac{r_{t-2}^{10} + r_{t-1}^{10} + r_t^{10}}{3}.$$

Expression of the horizontal capital

We consider a 1€ contribution paid at time $t = 0$ and the corresponding liabilities it produces $T = 40$ years later using the horizontal and vertical methods.

In the first case, the liability only depends on the initial rates and is deterministic:

$$L_T^h = e^{R_0 T} = \exp\left(\pi A T + \frac{\pi B T}{3}\left(r_{-2} + r_{-1} + r_0\right)\right),$$

where A and B are related to the yield rate of a 10-year ZC bond in the Vasicek model:

$$B = \frac{1}{k}\left(1 - e^{-10k}\right),$$

$$A = \left(\frac{\sigma^2}{2k^2} - \theta\right)(10B - 10) + \frac{\sigma^2}{4k}(10B)^2.$$

Expression of the vertical capital

The vertical liability is however a random variable, which, in the stochastic setting presented *supra*, is equal to

$$L_T^v = \exp\left\{\pi A T + \pi B\left(\frac{1}{3}r_{-2} + \frac{2}{3}r_{-1} + (1+\Lambda(1))r_0 + \theta(T-2-\Lambda(1))\right)\right.$$
$$\left. + \pi B\sigma \sum_{t=1}^{T-1} \Lambda(t) \int_{t-1}^{t} e^{ks} dW_s^r \right\},$$

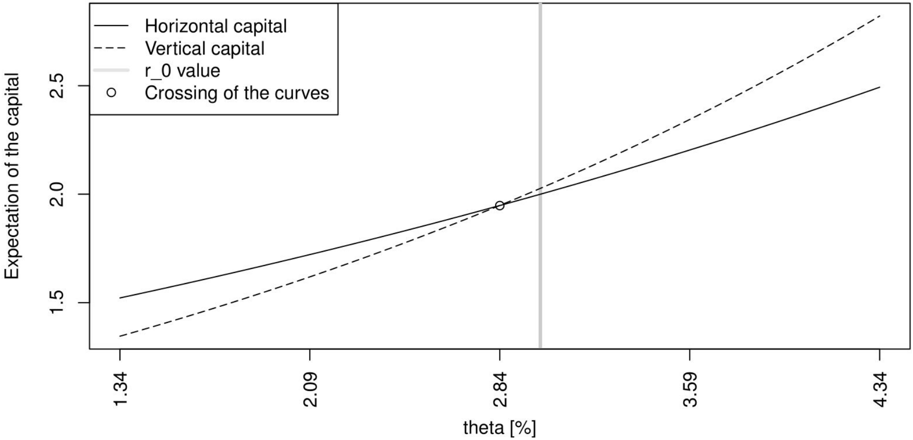
where

$$\Lambda(t) = \sum_{u=t}^{T-1} \lambda(u) e^{-ku}, \qquad \lambda(t) = \begin{cases} 1 & \text{if } t = 1, 2, \ldots, T-3 \\ \frac{2}{3} & \text{if } t = T-2 \\ \frac{1}{3} & \text{if } t = T-1 \end{cases}.$$

In particular, $L_T^v \sim \mathcal{LN}(m_v, s_v^2)$ for some m_v and s_v.

Expectations comparison (affiliate's view)

We first compare straightforwardly the expectations of the two liabilities.



The conclusion is consistent with the intuition we have gained from the very simple example: the horizontal method is better than the vertical one in a decreasing rate market and vice-versa.

ALM comparison (pension sponsor's view)

We now turn to another comparison method, and assume that the sponsor offering the pension scheme to the affiliated invests the paid contributions in an investment portfolio.

Three investments opportunities exist: a stock (x of the total), a rolling bond with fixed maturity (10 years, y of the total) and a bank deposit account (cash, $z = 1 - x - y$ of the total).

In order to compare the two methods, we compute the prices of:

1. The option giving the right to exchange the asset portfolio for the horizontal capital (a standard call option, as L_T^h is constant),
2. The option giving the right to exchange the asset portfolio for the vertical capital (an actual exchange option, as L_T^v is random).

The price of these options can be obtained using the well-known Margrabe formula (in a stochastic rates environment).

ALM comparison (pension sponsor's view)

We assume that the stock asset is a geometric Brownian motion, so that the total value of the portfolio in the risk neutral world follows the dynamics

$$\frac{dA_t}{A_t} = r_t dt + x\eta\sqrt{1-\rho^2}\,dW_t^S + (x\eta\rho - y\sigma 10B)\,dW_t^r,$$

where x and y are the proportion η is the volatility parameter of the stock asset, W^S and W^r are two standard Brownian motions and ρ is the correlation between them.

Thanks to the log-normal distribution of both the portfolio process and the vertical liability process, a closed formula can be obtained.

Expression of the price (vertical liability)

On one hand, the price of the option giving the right at time T to exchange the asset portfolio for the vertical liability is equal to

$$p_0^v = \Phi\left(\frac{-\left(\tilde{m}_v + \frac{\tilde{s}_v^2}{2}\right) + \frac{1}{2}\left(s_v^2 + s_a^2 - 2c_{v,a}\right)}{\sqrt{s_v^2 + s_a^2 - 2c_{v,a}}}\right) - \exp\left(\tilde{m}_v + \frac{\tilde{s}_v^2}{2}\right)\Phi\left(\frac{-\left(\tilde{m}_v + \frac{\tilde{s}_v^2}{2}\right) - \frac{1}{2}\left(s_v^2 + s_a^2 - 2c_{v,a}\right)}{\sqrt{s_v^2 + s_a^2 - 2c_{v,a}}}\right),$$

where $L_T^v \sim \mathcal{LN}(m_v, s_v^2)$, $A_T \sim \mathcal{LN}(m_a, s_a^2)$, $c_{v,a} = \text{covar}\left(\log L_T^v, \log A_T\right)$, and

$$\mathbb{E}_Q\left[\exp\left(-\int_0^T r_s ds\right) L_T^v\right] = \exp\left(\tilde{m}_v + \frac{\tilde{s}_v^2}{2}\right).$$

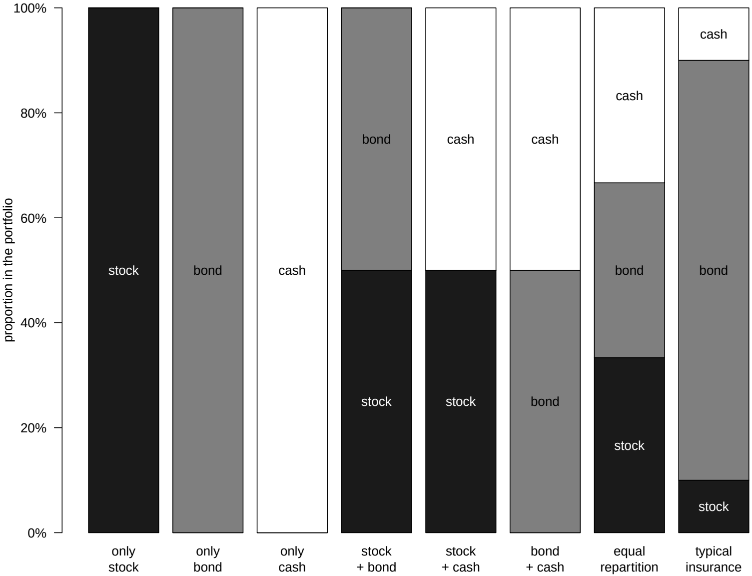
Expression of the price (horizontal liability)

On the other hand, the price of the option giving the right at time T to exchange the asset portfolio for the horizontal liability, which reduces to a standard call option on the asset portfolio with the strike being equal to the final value of the horizontal capital, is equal to

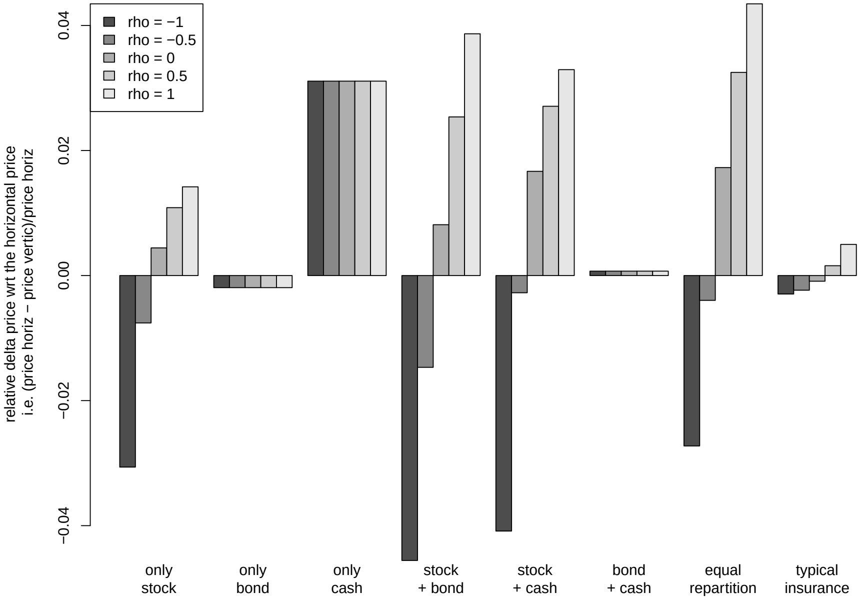
$$p_0^h = \Phi\left(\frac{\log \frac{1}{L_T^h\ P(0,T)} + \frac{1}{2}s_a^2}{s_a}\right) - L_T^h\ P(0,\,T)\ \Phi\left(\frac{\log \frac{1}{L_T^h\ P(0,T)} - \frac{1}{2}s_a^2}{s_a}\right),$$

where $P(0,\,T)$ is the price of a zero-coupon bond with maturity T.

Examples of portfolios



Results: relative delta prices



Interpretation of the results

Horizontal liability is cheaper when the portfolio is made of bonds, because this method « looks like » a bond.

On the contrary, vertical liability is cheaper when the portfolio is made of cash, because this method « looks like » cash.

When stocks are included in the portfolio, the hierarchy between the methods depends on the correlation between stocks and rates:

- When $\rho > 0$, the portfolio is close to cash, and the vertical method is preferred,
- When $\rho < 0$, the portfolio is very different from cash, and the horizontal method is preferred.

Conclusions

The two comparison methodologies give different points of view of the two computation methods.

They lead to different results depending on the market parameters and investment choices.

We have seen that they represent two different philosophies: the horizontal one is close the insurers' investment habits, while the vertical one is closer to pension funds' asset preferences.

Political circumstances have led Belgian authorities to let the choice for new pensions schemes... but as the results generated by the two methods are different, a whole set of juridical questions rise, among which discrimination problems (possible arbitrage by the sponsor).

Thank you for your attention!