A discount curve for Insurance Risk Management with exact fit and parsimonious forecasts

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Plan

1. Context
2. Curve construction and extrapolation
   - Curve construction
   - Curve extrapolation
3. Real world forecasting

1. Context

1. **Context**
2. Curve construction and extrapolation
   - Curve construction
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3. Real world forecasting

1. Context

- Calculation of insurers' technical provisions
- Discount curve **calibration** and **extrapolation** for pricing and risk neutral simulation
- Curves' **real world forecasting** for risk management

2. Curve construction and extrapolation

2.1. Curve construction

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2. Curve construction and extrapolation

2.1. Curve construction

- Discount curve construction: uses IRS + Credit Risk Adjustment (Solvency II framework) or OIS
- Curve construction methodology: ideas from Schlögl and Schlögl (2000)
- We add:
  - static discount curve **extrapolation**
  - curves' **real world forecasting**
- Can be extended to multiple curve construction

2. Curve construction and extrapolation

2.1. Curve construction (cont'd)

- **Idea**: No arbitrage short rate models include $t \mapsto b(t)$ for exact **fitting of initial term structure**
- **Examples**:
  - Hull & White (1990, 1994) extended Vasicek
  - Extended CIR...
- **piecewise-constant specification** of $t \mapsto b(t)$ at input swaps' maturities $T_1, \ldots, T_n \longrightarrow b_1, \ldots, b_n$

2. Curve construction and extrapolation

2.1. Curve construction (cont'd)

- **Discount factors** in the extended Vasicek case:

$$P(0,t) = exp\left(-X_0\phi(t) - a\int_{t_0}^{t} b(u)\phi(t-u)du - \psi(t)\right)$$

Where:

$$\phi(s) := \frac{1}{a}\left(1 - e^{-as}\right)$$

$$\psi(s) := -\int_0^s \left(\frac{\sigma^2}{2}\phi^2(s-\theta)\right) d\theta$$

- **Piecewise-constant** $t \mapsto b(t)$:

$$\int_0^t b(u)\phi(t-u)du \longrightarrow \sum_{k=1}^{n} b_k \int_{T_{k-1}\wedge t}^{T_k \wedge t} \phi(t-u)du \longrightarrow P^M(0,t)$$

2. Curve construction and extrapolation

2.1. Curve construction (cont'd)

- Calibration?
  - S = swaps values at time $t = 0$
  - C = swaps cash flows matrix
  - W = diagonal matrix of minimization weights
  - **P** = **unknown** discount factors, function of $b_1, \ldots, b_n$
- From **less bias (and more variance)** to **more bias (and less variance)**
  - Solve $S = C\mathbf{P}$ iteratively (a.k.a *bootstrapping*)
  - Minimize $(S - C\mathbf{P})^T W(S - C\mathbf{P})$
  - Minimize $(S - C\mathbf{P})^T W(S - C\mathbf{P}) + Penalization$

2. Curve construction and extrapolation

2.2. Curve extrapolation

2. Curve construction and extrapolation

2.2. Curve extrapolation

- In the Extended Vasicek case, for $t > T_n$ add a parameter:

$$b_{n+1} = f_\infty + \frac{\sigma^2}{2a^2}$$

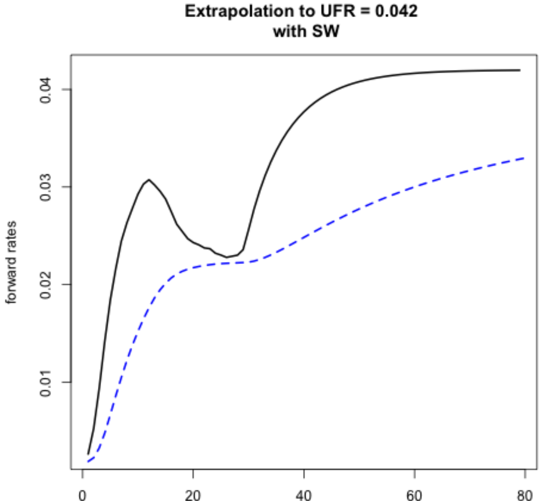
  where $f_\infty = UFR = \textit{Ultimate Forward Rate}$
- Either **exogenously fixed UFR** and convergence period $\tau_{convergence}$: increase parameter a until

$$|f^M(0, LLP + \tau_{convergence}) - f_\infty| < tolerance$$

- Or **data driven UFR**:
  - A fraction of the swaps data in a **training set**, the other in a **test set**
  - A grid search on values for $f_\infty \longrightarrow$ lowest pricing error on swaps from the **test set**

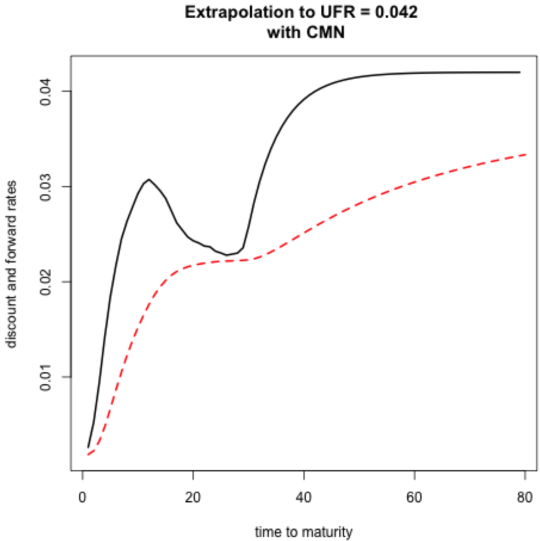
2. Curve construction and extrapolation

- IRS from Ametrano and Bianchetti (2013) + CRA = 10bps
- **Fixed UFR** = 4.2%, LLP = 20 years, convergence period $\tau_{convergence}$ = 40 years, **Smith-Wilson method**:



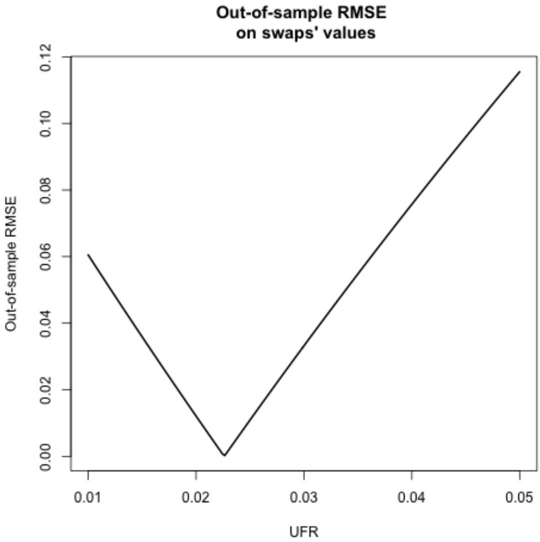
2. Curve construction and extrapolation

- **Fixed UFR** $= 4.2\%$, $LLP = 20$ years, convergence period $\tau_{convergence} = 40$ years, the **method presented here**:



2. Curve construction and extrapolation

- OIS from Ametrano and Bianchetti (2013), **data driven UFR** $= 2.26\%$:



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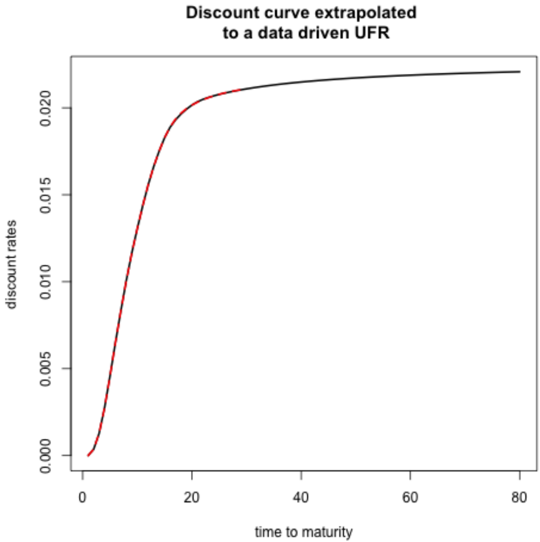

3. Real world forecasting

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3. Real world forecasting

- **Problem**: Forecasting the constructed ‘discretized HW’ curves to future dates, in real world probability
- **A solution**: problem’s **dimension reduction**
  - Functional Principal Components Analysis (**FPCA**) (Ramsay and Silverman (2005)) on parameters
  - Calibrate spot rates through time; obtain:

$$\mathbf{B}_{ij} = \hat{b}_{x_i}(t_j)$$

  - Based on the *cross-product function*:

$$V = \frac{1}{N}\mathbf{B}^T\mathbf{B}$$

  find Functional PCs: $\xi_1, \ldots, \xi_K$

3. Real world forecasting (cont'd)

- **A solution (cont'd)**:
  - Express **calibrated** $\hat{b}_{x_i}(t_j)$ as:

$$\hat{b}_{x_i}(t_j) = \beta_{t_j,0} + \sum_{k=1}^{K} \beta_{t_j,k}\xi_k(x_i) + \epsilon_{x_i}(t_j)$$

  - Obtain forecasts for $\hat{b}_{x_i}(t_j)$ at $t > t_j$: **univariate time series forecasts** on $\beta_{t_j,k}$

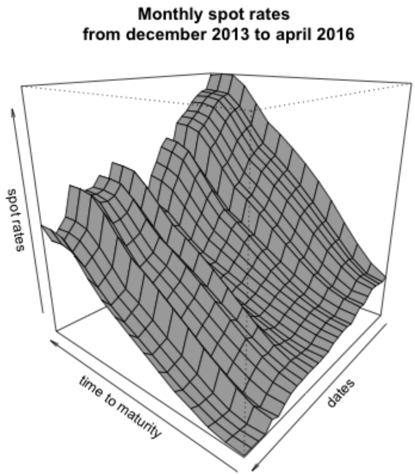
$$\beta_{t_j,k} \longrightarrow \hat{\beta}_{t_j+h|t_j,k}$$

3. Real world forecasting (cont'd)

- **A solution (cont'd)**:
  - Real world simulation: parametric law or **bootstrap resampling (with replacement)** of univariate time series **residuals**
  - Bootstrap:
    $$\hat{\beta}_{t_j,k} \longrightarrow \hat{\beta}^*_{t_j,k}$$
  - Obtain univariate forecasts:
    $$\hat{\beta}^*_{t_j+h|t_j,k}$$
  - And:
    $$\hat{b}^*_{x_i}(t_j + h)$$
  - Plug $\hat{b}^*_{x_i}(t_j + h)$ into formulas for discount factors

3. Real world forecasting (cont'd)

- IRS + CRA data. **12-months ahead forecasts**, **1000 simulations**, **FPCA** and **bootstrap resampling**:



3. Real world forecasting (cont'd)

- Benchmarking with models *à la* Diebold-Li (2006)
- Reasonable forecasts. But benchmarks are **subjective**
- In practice: study the univariate time series

Model	Avg. OOS error
CMN - `auto.arima`	0.0031
CMN - `ets`	0.0037
NS - `auto.arima`	0.0031
NS - `ets`	0.0035
NSS - `auto.arima`	0.0027
NSS - `ets`	0.0035

- Next: use of AR(1)

3. Real world forecasting (cont'd)

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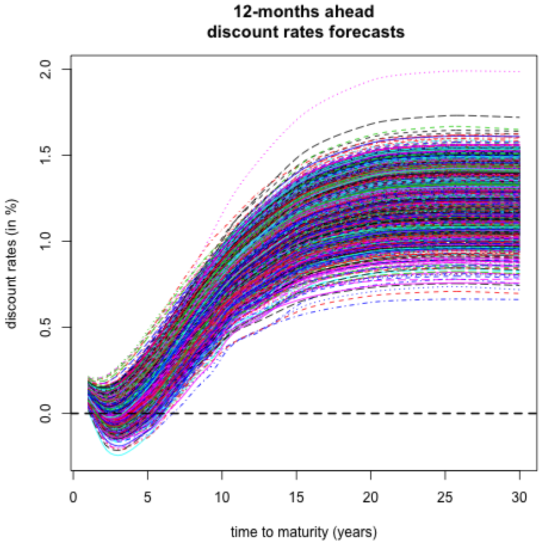

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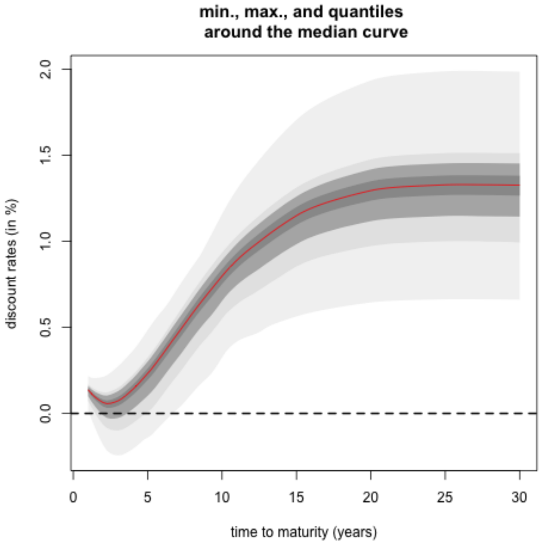

Questions

- Questions?

References

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